Sesión Especial 24

PDE fluids

Organizadores

- Francisco Gancedo (Universidad de Sevilla)
- Rafael Granero-Belinchón (Universidad de Cantabria)

Descripción

En estos últimos años ha habido un gran avance en el estudio matemático de problemas físicos que provienen de la dinámica de fluidos. En particular, el problema de la existencia global y la aparición de singularidades en tiempo finito ha sido investigado en profundidad y de manera exitosa por varios investigadores. Por otro lado, aún se resisten cuestiones fundamentales del área, con la existencia en tiempo finito de singularidades para modelos clásicos como Euler y Navier-Stokes. Este área se destaca por su pluridisciplinariedad, donde varias ramas de las matemáticas interaccionan. En esta sesión especial pretendemos reunir a matemáticos que trabajan en ecuaciones en derivadas parciales desde distintos puntos de vista con el propósito específico de fomentar la discusión y el intercambio de ideas.

Programa

Lunes, 4 de febrero (mañana)

12:00 – 12:30	Jon Asier Bárcena Petisco (Laboratoire Jacques-Louis Lions, Sorbonne Université) *Controlabilidad a cero de una ecuación de Stokes penalizada en dimensión dos con un control escalar*
12:30 – 13:00	Ángel Castro (ICMat-CSIC) *The Muskat problem in unstable regimes*
13:00 – 13:30	Jezabel Curbelo (Universidad Autónoma de Madrid) *Two-dimensional compressible convection at infinite Prandtl number*

Lunes, 4 de febrero (tarde)

17:00 – 17:30	Daniel Lear (ICMat-CSIC) *Global existence of quasi-stratified solutions for the confined IPM equation*
17:30 – 18:00	Sara Merino-Aceituno (University of Vienna/ University of Sussex) *Coupled Self-Organized Hydrodynamics and Stokes models for suspensions of active particles*

Martes, 5 de febrero (mañana)

11:30 – 12:00	Tania Pernas-Castaño (Hausdorff Center for Mathematics–Institute for Applied Mathematics) *El problema de Muskat no homogéneo*
12:00 – 12:30	David Poyato (Universidad de Granada) *Filippov flows in the singular Kuramoto model: from agent-based to fluid descriptions*
12:30 – 13:00	Stefano Scrobogna (Basque Center for Applied Mathematics) *Asymptotic models for free boundary Darcy flows*

Controlabilidad a cero de una ecuación de Stokes penalizada en dimensión dos con un control escalar

Jon Asier Bárcena Petisco

Laboratoire Jacques-Louis Lions, Sorbonne Université

barcena@ljll.math.upmc.fr

Resumen. En esta charla consideramos una ecuación de Stokes penalizada en un abierto regular y acotado $\Omega \subset \mathbb{R}^2$ y con condiciones fronteras del tipo Dirichlet. Demostramos que, mediante un control escalar que actúa en un pequeño abierto en el interior de Ω, podemos controlar a cero a nuestro sistema, y además con un coste del control uniforme con respecto al parámetro que tiende a cero. El control se obtiene mediante desigualdades de Carleman y estimaciones elípticas que demostraremos.

Este trabajo se ha realizado gracias a un contrato doctoral financiado por la Región Ile-de-France.

The Muskat problem in unstable regimes

Ángel Castro

ICMat-CSIC

angelcastro@icmat.es

Resumen. In this talk we will consider the Incompressible Porous Media equation with an initial data of Muskat type in the unstable regime. After discussing the physics of the problem, we will show how the covex integration allow us to construct solutions of mixing type in this situation in which the classical Muskat equation is ill-posed. Also, we will present some new results addresses to the construction of solutions in the partial unstable regime

---

Two-dimensional compressible convection at infinite Prandtl number

Jezabel Curbelo

Universidad Autónoma de Madrid

jezabel.curbelo@uam.es

Abstract. We study infinite Prandtl number convection using an ideal gas equation of state. For this purpose, we developed a numerical method for the set of equations governing fully compressible convection. Reduced models have also been analyzed, such as the anelastic approximation and the anelastic liquid approximation. Specific effects arising because of the compressibility of the fluid are studied, like the scaling of viscous dissipation and the scaling of the heat flux contribution due to the mechanical power exerted by viscous forces. We analyzed the solutions obtained with each model (full compressible model, anelastic and anelastic liquid approximations) in a wide range of dimensionless parameters and determined the errors induced by each approximation with respect to the full compressible solutions. Based on the different forms of entropy balance between exact and anelastic models, we find that a necessary condition for convergence of the anelastic results to the exact solutions is that the product of the ratio of the superadiabatic temperature difference to the adiabatic difference with the ratio of the superadiabatic heat flux to the heat flux conducted along the adiabat must be small compared to unity.

Joint work with Thierry Alboussiere , Stephane Labrosse , Yanick Ricard, Fabien Dubuffet and Lucia Duarte

---

Global existence of quasi-stratified solutions for the confined IPM equation

Daniel Lear

ICMat-CSIC

daniel.lear@icmat.es

Abstract. In this talk, we consider a confined physical scenario to prove global existence of smooth solutions with bounded density and finite energy for the inviscid incompressible porous media (IPM) equation. The result is proved using the stability of stratified solutions, combined with an additional structure of our initial perturbation, which allows us to get rid of the boundary terms in the energy estimates.

Joint work with Ángel Castro and Diego Córdoba

Coupled Self-Organized Hydrodynamics and Stokes models for suspensions of active particles

Sara Merino-Aceituno

University of Vienna/ University of Sussex

sara.merino@univie.ac.at

Abstract. We derive macroscopic dynamics for collective motion in a fluid. The starting point is a coupled Vicsek-Stokes system. The Vicsek model describes self-propelled agents interacting through alignment. It provides a phenomenological description of steric interactions between agents at high density. Stokes equations describe a low Reynolds number fluid.

Joint work with Pierre Degond (Imperial College London), Fabien Vergnet (Universit?e Paris-Sud), Hui Yu (Tsinghua University)
Financiado por WWTF (Vienna Science and Technology Fund)

El problema de Muskat no homogéneo

Tania Pernas-Castaño

Hausdorff Center for Mathematics–Institute for Applied Mathematics

pernas@iam.uni-bonn.de

Resumen. El problema de Muskat no homogéneo estudia la evolución de la interfaz que se crea entre dos fluidos de diferente naturaleza cuando se encuentran en un medio poroso con un salto de permeabilidades. La velocidad del fluido en un medio poroso viene modelada por la ley experimental conocida por ley de Darcy:

$$\frac{\mu}{\kappa}u = -\nabla p - (0, g\rho),$$

donde $(x,t) \in \mathbb{R}^2 \times \mathbb{R}^+$, $u = (u_1(x,t), u_2(x,t))$ es la velocidad del fluido incompresible $(\nabla \cdot u = 0)$, $p = p(x,t)$ es la presión, $\mu = \mu(x,t)$ es la viscosidad dinámica del fluido, $\kappa = \kappa(x)$ es la permeabilidad del medio, $\rho = \rho(x,t)$ es la densidad del fluido y g es la aceleración debido a la gravedad. En esta charla trataremos la existencia local en espacios de Sobolev H^k para $k \geq 3$ de la frontera libre, cuando esta se crea por la discontinuidad entre las densidades y viscosidades de los fluidos:

$$(\mu, \rho)(x,t) := \begin{cases} (\mu^1, \rho^1) & x \in \Omega^1(t) \\ (\mu^2, \rho^2) & x \in \Omega^2(t) = \mathbb{R}^2 - \Omega^1(t) \end{cases},$$

y $\kappa(x)$ es una función de salto que separa dos regiones con diferentes valores de permeabilidad:

$$\kappa(x_1, x_2) := \begin{cases} \kappa^1 & x \in \Omega_1(t) \cup \Omega_2(t) = \mathbb{R}^2 - \Omega_3, \\ \kappa^2 & x \in \Omega_3. \end{cases}$$

donde μ^1, μ^2, ρ^1, ρ^2, κ^1 y κ^2 son constantes (Ver [1]). Esto nos permitirá adentrarnos en el estudio de singularidades a tiempo finito, de tipo splash y splat (ver [2]).

Referencias

[1] Tania Pernas-Castaño. Local-existence for the Inhomogeneous Muskat problem. *Nonlinearity*, 30(5):2063, 2017.
`http://iopscience.iop.org/article/10.1088/1361-6544/aa6691/meta`

[2] A. Castro, D. Córdoba, C. Fefferman, F. Gancedo and J. Gómez-Serrano. Finite time singularities for the free boundary incompressible Euler equations. *Ann. of Math.*(2), 178(3):1061–1134, 2013.

Trabajo en colaboración con Diego Córdoba
Financiado por ICMAT Severo Ochoa project SEV-2015-556 and the grant MTM2014-59488-P (Spain)

Filippov flows in the singular Kuramoto model: from agent-based to fluid descriptions

David Poyato

University of Granada

davidpoyato@ugr.es

Abstract. The classical Kuramoto model was proposed as the prototype system modeling sincronization of N agents that interact through periodic rules. Although discrete agent-based models are interesting by themselves, real life situations involve a large amount N of agents, which are modeled by systems of N coupled ODEs. In some cases, we can obtain an approximate dynamics with an only PDE that governs the macroscopic/fluid description of the system.

In this talk we will focus on the Kuramoto model with non-uniform and singular weights. First, we shall introduce the agent-based system of N coupled oscillators and three associated regimes of singularity: subcritical, critical and supercritical. We will propose a well-posedness theory in the sense of Filippov to face the presence of singularities, giving rise to solutions with new rich behavior: finite-time phase synchronization and clustering into distinguished groups. Later, we will introduce the associated macroscopic Vlasov equation. It consists in a fluid-type model governed by a conservation law for the probability density of oscillators along the manifold $\mathbb{T} \times \mathbb{R}$, where the (compressible) velocity field is nonlocal and self-generated. Since the kernel is singular, we will propose a well posedness theory via the concept of weak measure-valued solutions in the sense of the Filippov flow. Such solutions emerge as rigorous mean field limit when the number N of particles tends to infinity. Finally, we will conclude by remarking some analogies and differences with other related models in the literature like the singular Cucker-Smale model.

Referencias

[1] J. Park, D. Poyato, J. Soler, Filippov trajectories and clustering in the Kuramoto model with singular couplings. arXiv:1809.04307.

[2] D. Poyato, Filippov flows and mean-field limits in the kinetic singular Kuramoto model. *Preprint.*

Joint work with Jinyeong Park (Hanyang University, Korea) and Juan Soler (University of Granada, Spain).
This work has been partially supported by the MECD (Spain) grant FPU14/06304, the MINECO-Feder (Spain) grant MTM2014-53406-R and the Junta de Andalucía (Spain) Project FQM 954.

Asymptotic models for free boundary Darcy flows

Stefano Scrobogna

Basque Center for Applied Mathematics

sscrobogna@bcamath.org

Abstract. In the first part of the talk I will provide rigorous asymptotic models for the free boundary Darcy problem under the assumption of weak nonlinear interaction, in a regime in which the steepness parameter of the interface is considered to be very small. The second part of the talk will be devoted to prove some rigorous result of well-posedness in critical spaces for the asymptotic model.

Joint work with R. Granero-Belinchón.
